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Chaos Detection and Predictability

Springer

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Preface

Being able to distinguish chaoticity from regularity in deterministic dynamical systems, as well as to specify the subspace of the phase space in which instabilities are expected to occur, is of utmost importance in as disparate areas as astronomy, particle physics, and climate dynamics. The presence of chaos introduces limitations in our ability to accurately predict the evolution of a dynamical system at scales of different sizes. In many practical applications it is of great importance to determine the significance of this effect to the overall dynamics of the system. For this reason, the development of precise and efficient numerical tools for distinguishing between order and chaos, both locally and globally, becomes imperative, especially in the case of multidimensional systems, whose phase space is not easily visualized. Nowadays there exists a plethora of such methods.

The workshop "Methods of Chaos Detection and Predictability: Theory and Applications" which was held in June 2013 at the Max Planck Institute for the Physics of Complex Systems, in Dresden, Germany, brought together specialists who have developed such methods, as well as researchers applying those techniques to a variety of problems in the natural sciences. This book reviews the theory and numerical implementation of several of the existing methods of chaos detection and predictability and presents the current state of the art. Its chapters are written by the creators of these methods and/or by well-established experts included in the workshop's list of invited speakers.

The most commonly employed method for investigating chaotic dynamics is the computation of the Lyapunov Exponents (LEs). These are asymptotic measures characterizing the average rate of growth (or shrinking) of small perturbations to the orbits of a dynamical system, with the positivity of the maximum LE (mLE) indicating chaoticity. The basic concepts of LEs are presented in the first chapter of the book written by U. Parlitz, where the particular case of the LEs' estimation for time series is discussed and analyzed in depth.

As successful and illuminating LEs have been to characterize chaoticity in deterministic dynamical systems, they suffer in certain situations from serious drawbacks: For example, their computed values can vary significantly in time and may only be used in the long time limit when the exponents have converged with satisfactory accuracy. Furthermore, in the case of (noisy) experimental data, they rely on phase space reconstruction methods, whose inattentive implementation might produce unreliable results.

In the last two decades, several methods have been developed for the fast and reliable determination of the regular or chaotic nature of orbits which were aimed to surmount the shortcomings of the traditional methods involving LEs and phase space reconstruction. These methods can be divided in two broad categories: those which are based on the study of the evolution of deviation vectors from a given orbit, like the computation of the mLE, and those which rely on the analysis of the particular orbit itself.

A technique closely related to the computation of the mLE, which exploits the information provided by the short time evolution of a deviation vector, is the Fast Lyapunov Indicator (FLI) discussed in the second chapter of the book by E. Lega, M. Guzzo, and C. Froeschlé. The next chapter by R. Barrio deals with some variants of the FLI method, namely, the Orthogonal Fast Lyapunov Indicator (OFLI and OFLI2). The method of the Mean Exponential Growth factor of Nearby Orbits (MEGNO), which again is based on the evolution of one deviation vector from the reference orbit, is presented in the next chapter by P. Cincotta and M. Giordano.

The utilization of more than one deviation vector for the characterization of chaos is considered in the next chapter by Ch. Skokos and Th. Manos where the methods of the Smaller (SALI) and the Generalized Alignment Index (GALI) are presented. The method of the Relative Lyapunov Indicator (RLI) where the differences of the finite-time estimators of the mLE of two nearby orbits are used to characterize chaos is the content of the next chapter by Z. Sándor and N. Maffione.

In the following chapter by G. Gottwald and I. Melbourne, the “0-1” test for chaos is discussed in detail. Contrary to the five previous chapters, the analysis in the “0-1” test for chaos is performed directly on the actual orbit (or time series).

The presence of chaos and a positive mLE is often seen as a limitation to the predictability time of the underlying system, which is crudely estimated to be inversely proportional to the mLE (the so-called Lyapunov time). The situation in complex systems evolving on several temporal scales, like for example, in weather forecasting models, can be, however, much more intricate as is shown in the last chapter of the book by S. Siegert and H. Kantz: reliable predictions can be made for times much longer than suggested by the predictability horizon implied by the Lyapunov time.

We hope that this book will be useful both for young scholars, like graduate students, Ph.D. candidates, and postdocs, and for specialists aiming at an up-to-date review of some of the most widely used techniques of chaos detection and predictability.

We thank the Springer Editorial Board, the authors, as well as the reviewers of all chapters, for their work and effort which made possible the publication of this volume.

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The original version of the book was revised: The first editor's name was corrected.

The publisher apologizes for having published an early draft edition of the preface. This has been updated to the final version. The Erratum to the book is available at DOI 10.1007/978-3-662-48410-4_9

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